

# Turbulence in Plasma-Induced Hypersonic Drag Reduction

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With the use of a pressure-ruptured shock tube, an experimental study of the dynamics of shock waves in weakly ionized argon, krypton, xenon, and neon glow discharge plasmas has been performed. For Mach numbers in the range 1.5–3, our results show that there is an increase in the velocity of a shock wave when it makes a transition from a neutral gas to a weakly ionized plasma, and the effect increases with increasing atomic weight. The observations indicate that the shock acceleration cannot be accounted for by thermal effects. However, the use of turbulence models based on reduced kinetic theory and second-order phase transitions suggests a consistent role for turbulence in plasma-induced hypersonic drag reduction.

## I. Introduction

PREVIOUS investigators have suggested that for weakly ionized gases under a variety of circumstances,<sup>1–3</sup> an increase in the shock wave velocity and a corresponding decrease in shock amplitude can result compared to the un-ionized case. The decreased shock amplitude presents a means of improving the flight characteristics at high altitude by weakly ionizing the gas through which the flight occurs. The decreased shock strength in this medium then offers a substantial potential influence on the drag forces on the vehicle.

The anomalous dynamics of shock waves in weakly ionized glow discharge plasmas have been studied experimentally in both nonequilibrium molecular gases, such as air and nitrogen, and monatomic gases such as argon and xenon.<sup>4,5</sup> In these experiments, plasmas formed in gases at a pressure of 15–30 torr (1999.8–3999.67 Pa) were used. The plasma electron temperature was typically a few electron volts and the degree of ionization of the order of  $10^{-6}$ – $10^{-5}$ . In some cases, a magnetic field was applied either longitudinal or transverse to the direction of the shock wave to study the role of the charged species on shock dynamics. The results of these experiments revealed similar features. Specifically, anomalously high shock velocities were observed in the plasma medium, together with a significant dispersion and weakening of the shock wave.

There is a growing consensus that most cases of plasma-based shock wave acceleration in supersonic flow are a consequence of plasma heating. However, there is now evidence to support a role for turbulence in shock wave drag reduction and the potential opportunity of shock wave drag reduction at low plasma concentrations.<sup>6</sup> We present the results of a series of experiments looking primarily at the acceleration of shock waves in weakly ionized argon, krypton, xenon, and neon plasmas. A glow discharge plasma was used, with an electron density  $n_e$  of  $10^{10}$  cm<sup>−3</sup>, neutral density  $n_0 \sim 10^{16}$  cm<sup>−3</sup>, and electron temperature  $T_e \sim 1$ – $3$  eV. The preplasma shock speeds ranged from about 500 to 950 m/s. We examine the extrapolation of the previously observed effects of a connection between turbulence and shock wave drag reduction in argon to a broader class of weakly ionized gases and look for systematic trends.

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## II. Experimental Procedures

The shock waves were produced in a cylindrical shock tube as indicated in Fig. 1. The driver section is 60 in. (1.524 m) long with an i.d. of 5.2 in. (0.132 m); the pressure in this section was varied from 200 to 500 torr using research grade air as the driver gas. The driven section begins with a transition piece that reduces the 5-in. (0.127-m) i.d. to a 3-in. (0.076-m) i.d. test section. The shock waves produced in this fashion are consistent with the standard equations of one-dimensional shock tube flow. In the test section, we used the following noble gases: argon (atomic weight 39.9 g, density 1.78 g/l, and viscosity 22.9  $\mu$ P, for conditions in this experiment), krypton (atomic weight 83.80 g, density 3.73 g/l, and viscosity 25.6  $\mu$ P), neon (atomic weight 20.2 g, density 0.89 g/l, and viscosity 32.1  $\mu$ P), and xenon (atomic weight 131.29 g, density 5.89 g/l, and viscosity 23.2  $\mu$ P). The local speeds of sound in these gases at 338 K are 332, 280, 465, and 223 m/s, respectively.

The test section is made up of three 12-in. (0.305-m)-long modular quartz tubes. Five Kistler 606A piezoelectric pressure transducers situated flush with the inside walls and spaced 5.5 in. (0.1397 m) apart along the tubes allowed the speed of the shock wave to be measured. Signals were recorded using two Tektronix 784D digitizing oscilloscopes, the first of which is triggered by a pressure transducer located in the high-pressure section. Two channels on this oscilloscope record the pressure in a region where there is no plasma; two others record the pressure in the plasma region. Samples of the pressure transducer data are shown in Fig. 2. The overall time for the data shown is 300  $\mu$ s. The second oscilloscope recorded the plasma spectral information as a function of time.

The driven section is filled with the test gas at 1 torr. We continuously replenish the flow of gas through this section by running the vacuum pump at a very low speed and leaving the loading valve slightly open. The glow discharge plasma is produced in the central quartz tube between hollow molybdenum electrodes with a 2-kV dc constant current source (Universal Voltronics BRC 20000) using currents ranging from 60 to 200 mA. As discussed in previous work,<sup>6</sup> the degree of ionization of the plasma,  $\alpha = n_i/n_0$  (at fixed pressure) can be controlled by varying the mean discharge current. (Here  $n_i$  is the plasma ion density and  $n_0$  is the neutral gas density.) The on-axis plasma gas temperature is monitored (before the shock wave is produced) with a chromega/alomega thermocouple junction situated 60 mm from the positive electrode.

Once a quasi-steady plasma is created at the desired pressure, a shock wave is generated by breaking the diaphragm. The shock travels downstream past the first two pressure transducers in the first quartz tube. The reading of these transducers gives the speed of the shock wave before it enters the plasma. The transducers on the plasma-filled central quartz tube give the shock speed in the plasma. Two electric probes each with a 1-mm-diam, 1-mm-long probe tip located in the middle of the plasma section are used to measure plasma properties such as electron number density  $n_e$  and electron temperature  $T_e$ . Optical spectral radiation from the plasma is observed with a diffraction grating Acton Research Corp. (ARC) Spectrapro-5001 spectrometer and photomultiplier station.

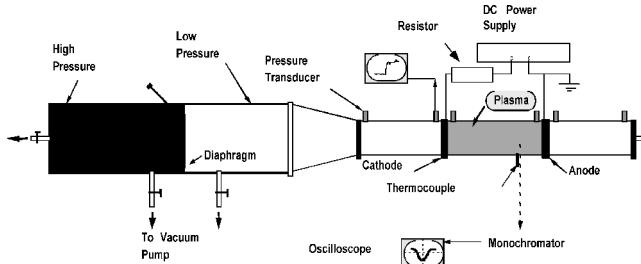


Fig. 1 Schematic of the shocked plasma tube.

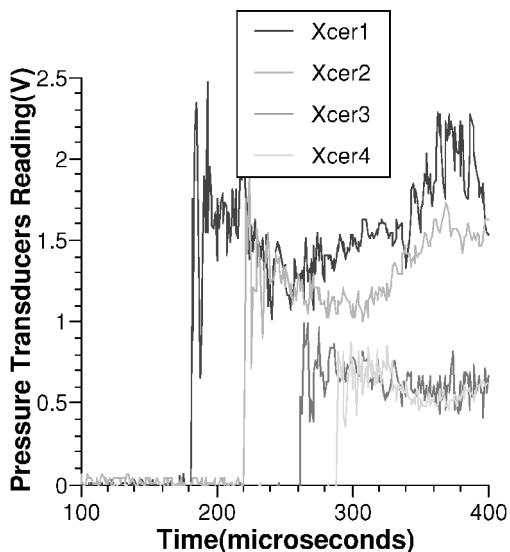


Fig. 2 Sample pressure data: The overall time for the data shown is  $300 \times 10^{-6}$   $\mu$ s.

The spectral measurements provide information on the ionization state of the plasma, as well as the emission characteristics of the plasma during the passage of the shock. When these diagnostics are used, the effect of the plasma on the shock wave can be characterized as a function of the plasma properties  $n_0$ ,  $n_e$ ,  $T_e$ ,  $\alpha$ , and shock Mach number. Because the local density, speed, and viscosity are known, we can determine the unit Reynolds number ( $Re/l$ ) for all of our data.

The argon experiments were performed over a range of shock Mach numbers from 1.5 to 3.0, using a test section pressure of either 1 or 4 torr (133.32 or 533.29 Pa) argon. [The highest pressure at which the discharge could be sustained was  $\sim 4.5$  torr (599.95 Pa).] For these measurements, the discharge current was varied between 20 and 200 mA, using a voltage of about 1 kV across the electrodes. After initiation, the plasma was allowed to stand for about 2 min to reach a quasi-steady state before the shock wave was generated. This time period was arrived at after an observation of the thermocouple readings over a period of several minutes. A number of experiments were performed with and without the plasma under identical conditions. In all cases, the propagation of the shock was monitored from the time it was generated until it was reflected from the end wall. The average speeds of the shock wave in the neutral gas and in the plasma were obtained to an accuracy better than 5% from the pressure jump of the transducer signals. When the plasma was used, the discharge current during the transit of the shock was measured using a potential divider connected to one of the channels of the oscilloscopes.

For neon, krypton, and xenon, the range of operating parameters was narrowed. The current in all of these cases was kept at 200 mA, and the pressure in the driven section was kept at 1 torr. For these data, we isolate trends associated with changes in current and trends associated with changes in the driven section pressure from trends solely derived from changes in the local Mach number (and, by extrapolation, the local temperature).

The preshock plasma current and optical emission signals obtained from the photomultipliers show that the plasma was quenched

by the interaction with the shock. This quenching was attributed to the interruption of the discharge current resulting from the sustained recombination and high-pressure effects that occurred when the shock passed through the plasma. The plasma decay time was characterized by the electron-folding time of the discharge current signals. Observations showed that for preplasma shock speeds of up to  $\sim 850$  m/s, that is,  $\sim$  Mach 2.7, the transit time in the plasma tube was less than the existence time of the 1-torr (133.32-Pa) plasma. This suggests that between the shock detecting transducers on the plasma tube, the shock wave interacted fully with a weakly ionized plasma. The decay times were observed to be shorter for the 4-torr (533.29-Pa) plasmas, on the order of 70  $\mu$ s, compared with transit times of about 200  $\mu$ s, within the plasma tube.

The plasma electron number density  $n_e$  and electron temperature  $T_e$  were measured using either a single-electrostatic (langmuir) probe or the double electrostatic probe method.<sup>7</sup> The measurements were performed near the midpoint of the positive column of the glow discharge. The results were as follows:  $n_e \sim 0.75 \times 10^{-10}$   $\text{cm}^{-3}$  and  $T_e = 3$  eV for the 1-torr (133.32-Pa) plasma and  $n_e = 10^{-10}$   $\text{cm}^{-3}$  and  $T_e = 1.5$  eV for the 4-torr (533.29-Pa) plasma. A study of the plasma radiation spectrum from 3000 to 7000  $\text{\AA}$  indicated that the plasma was predominantly singly ionized. This is in agreement with the results of the plasma density and temperature measurements. The degree of ionization of the plasma,  $\alpha$ , is, therefore, of the order of  $10^{-6}$ . The plasma gas temperature, which was measured with the thermocouple, peaked at  $\sim 330$  K on the axis for the 1-torr (133.32-Pa) discharge, falling to  $\sim 310$  K at a radius of 1 in. (0.0254 m) from the axis. At 4 torr (533.29 Pa), the plasma was observed to have a somewhat hollow structure near the positive electrode. In this case, the observed temperature on the axis was 330 K, dropping to 325 K at a radius of 1 in. (0.0254 m).

Spectral measurements were performed by observing the on-axis plasma with the spectrometer and photomultiplier tube arrangement. The data were digitized by a detector interface system (supplied by ARC) in combination with a personal computer. The evolution of selected spectral line relative intensity in real time was monitored as the shock wave goes through the plasma. The selected lines are as follows: for argon, Ar I 6965  $\text{\AA}$ ; for krypton, Kr I 5871  $\text{\AA}$ ; for neon Ne II 3574.6  $\text{\AA}$ ; and for xenon, Xe I 4916  $\text{\AA}$ . A pressure transducer signal was recorded together with the spectral signals to provide a timing fiduciary. A sample of such results is shown in Fig. 3.

The light fluctuations are studied for evidence of turbulent signatures. In these measurements, we perform Fourier analyses of the light fluctuations and look for the standard profile<sup>8</sup> of 1) power input range, 2) Kolomogorov-like decay, 3) energy characteristic input range, 4) fast decay, and 5) noise. In the Kolomogorov range, the power law decay is expected to be  $P \sim \omega^{-n}$ , where  $n$  is defined as the spectral index and is typically found in the range  $1 < n < 7$ . A sample of the data from xenon fluctuations is shown in Fig. 4. The fluctuation power spectrum is derived from 20,000 samples at 10 ns per sample. All analyses are done in Mathematica. The spectral index  $n$  is determined from data showing a Kolomogorov-like decay.

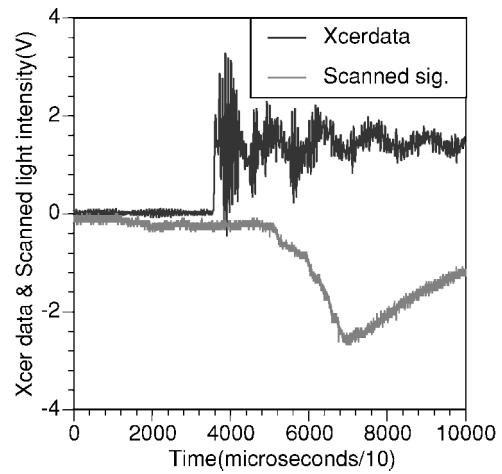
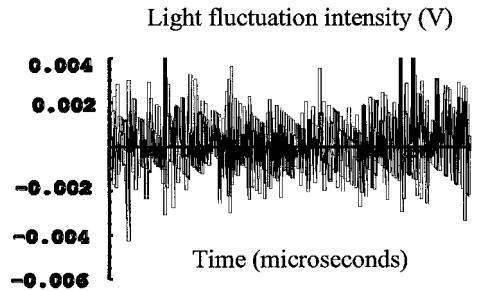


Fig. 3 Sample: pressure and light data. The light emissions are from excited argon neutrals.



Time (microseconds)

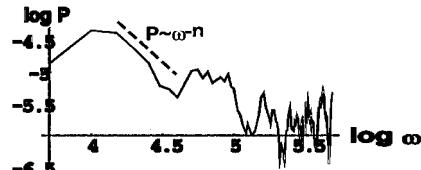
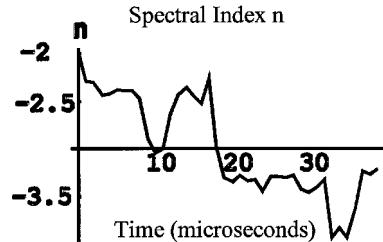
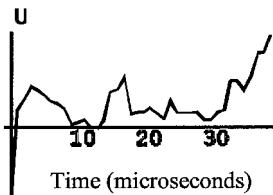


Fig. 4 Sample: light fluctuations and a power spectrum.



Turbulent Fluctuation Energy U

Fig. 5 Sample data: spectral index  $n$  and turbulent fluctuation energy  $U$  vs time.

In Fig. 5, a sample of the measured evolution of turbulence-related parameters is shown. The history of the spectral index  $n$  over 30  $\mu$ s is obtained from the Fourier analyses of 1000 measurements of light emission at a sample rate of 10 ns per sample. Therefore, each set represented 10  $\mu$ s of overlapping data. In each set, the total turbulent fluctuation energy  $U$  is calculated by integrating the power spectrum over the range  $0 < \omega < 200$  kHz. Figure 5 shows the evolution in  $U$  over 30  $\mu$ s for the data corresponding to the evolution in  $n$ .

### III. Results and Discussion

In Fig. 6, which includes all of the data collected on the argon plasma, the results of observations of shock speeds are given for 1- and 4-torr argon over a range of Mach numbers. We show the difference in shock wave speed between plasma and nonplasma flow vs Reynolds number  $Re/l$  for our measurements. There is an onset of Reynolds number-based influence on shock wave speed at roughly  $(Re/l) = 2 \times 10^3$  /m. In both cases, compared to the un-ionized gas, there is a significant increase in the shock speed when the shock wave makes a transition from the neutral gas to the weakly ionized plasma. The percentage increase in shock speed is higher for lower Mach numbers. At Mach  $2.0 \pm 0.2$ , the average percentage increase in the preplasma shock speed is about 14% for 1-torr plasma and 20% for 4-torr plasma. The largest increase for the measurements reported here was obtained at  $\sim 33\%$  in 4-torr plasma.

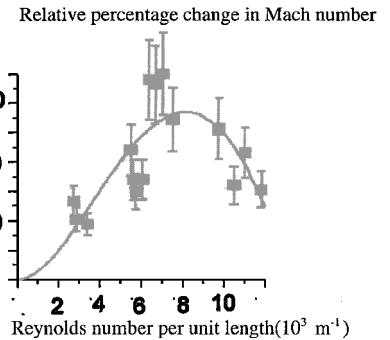


Fig. 6 Velocity enhancements: argon. The Mach number of the shock wave is determined before and after interacting with the argon plasma.

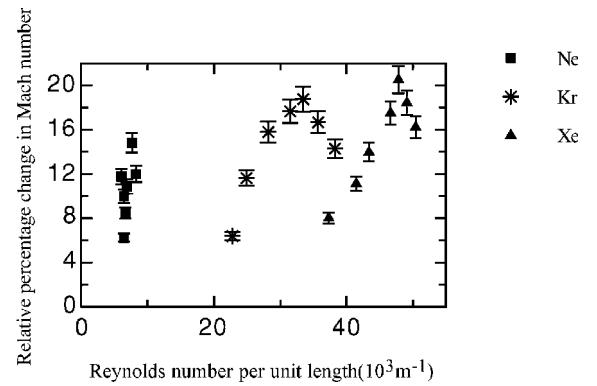


Fig. 7 Velocity enhancements: neon, krypton, and xenon.

The increase in shock speed did not correlate in any way with the gas temperature measured with the thermocouple for either 1- or 4-torr argon. There was also no observed dependence on the discharge current. However, it was not possible, using the current apparatus, to change the magnitude of the plasma discharge current by a large factor to observe a dependence on current.

Behavior similar to that in Fig. 6 is observed in all cases, for example, as in Fig. 7. Figure 7 is restricted to plasmas where the discharge current is 100 mA and the pressure in the test section is 1 torr. In all cases, there is a growth in importance of Reynolds number as a correlation with changes in the enhancement of local Mach number over a range of local Reynolds numbers. There is a peak value for the speed enhancement (associated with a peak value for Reynolds number  $Re/l$ ) followed by a roll off to lower values of speed enhancement. However, in all cases, it is difficult to infer an onset Reynolds number equivalent to that for argon.

By the use of reduced kinetic theory,<sup>9</sup> turbulent processes with correlation timescales comparable to that of a nonequilibrium process may distort the natural evolution of the nonequilibrium process.<sup>10</sup> The distortion has been observed in  $\text{NO}_2$  recombinations,<sup>11</sup> in polymer mixing,<sup>12</sup> in detonation waves,<sup>13</sup> and in heterogeneous nucleation.<sup>14</sup> In most cases just cited, the indication of turbulent distortion came in the form of a dependence of a turbulence sensitive parameter (droplet size, detonation reactant, etc.) on Reynolds number showing an Orr-Sommerfeld-like relationship (see Ref. 15) parameterized by a dimensionless characteristic frequency. Distortions in nonequilibrium processes can be manifest in the effective local speed of sound,<sup>16</sup> which is implicitly dependent on the molecular weight of the flow constituents. If plasma-induced drag reduction, resulting from a manipulation of the local speed of sound, is to be a turbulence sensitive parameter, then there may be a similar dependence of drag reduction on Reynolds number. The overall shape of the dependence shown and the atomic weight sensitivity in Figs. 6 and 7 are consistent with the predictions in Refs. 15 and 9. These results are at least qualitatively consistent with an important role for turbulence in drag reduction in hypersonic flow using weakly ionized gases.

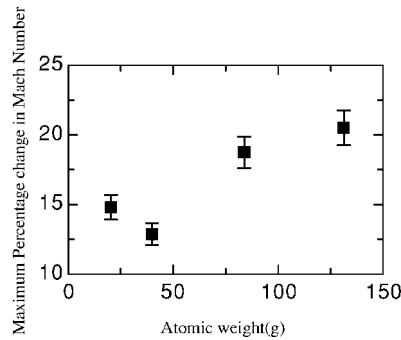


Fig. 8 Maximum percentage change in Mach number vs plasma atomic weight.

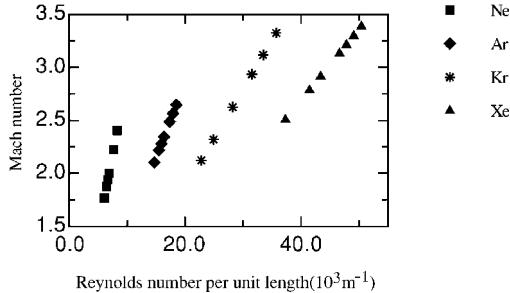


Fig. 9 Mach number range for turbulent influences vs corresponding Reynolds number.

The measurements on shock wave speed enhancement are summarized in Fig. 8. The peak Mach number enhancement is designated as  $\% \Delta M_c$ , where the subscript  $c$  indicates the measured value of the maximum percentage Mach number enhancement. The linearlike correlation between the peak Mach number enhancement and atomic weight is indicated. Because the value of  $\% \Delta M_c$  is definable (by measurement) for each atomic species, data in Fig. 8 suggest, by the correlation with atomic weight, that it might be derivable from fundamental microscopic parameters.

The data in Figs. 6 and 7 define regimes where turbulence produces a Mach number enhancement. Because each measurement is associated with a Reynolds number and each measurement is made at a specific value of primary shock wave Mach number, there could be parametrically an implicit correlation between the Mach number at which the enhancement effect is observed and its associated Reynolds number. This is particularly likely because, by keeping the glow discharge current and the glow discharge pressure constant, the effect of increasing Mach numbers (and thereby increasing the local temperature) should be isolated. In Fig. 9, the data for Mach number vs Reynolds for the turbulence-driven Mach number enhancement are shown, and a correlation does indeed seem to exist. For each species in our study, the turbulence sensitive regime seems to be well defined.

In addition, for each species, the nature of the correlation evidenced in Fig. 9 seems to be sensitive to the changes in atomic weight. Specifically, the slope defined as  $d(M)/d(Re)$  decreases with increasing atomic weight (as shown in Fig. 10). Notice, from Fig. 9, that the overall range of Mach numbers over which the effect is observed increases with increasing atomic weight. Notice further (from Fig. 10) that the Reynolds range over which the effect will be observed also increases with increasing atomic weight.

An additional interpretation of the shapes in Figs. 6 and 7 comes from associating the values of  $\% \Delta M_c$  with an unspecified transport parameter. This interpretation then affords the view of the evolutions as evidence of a lambda-like second-order phase change<sup>17</sup> in the underlying transition to turbulence. This interpretation characterizes transition to turbulence as a disorder to order transition, with increasing internal energy, specifically allowed in a Ginzburg-Landau effect (see Ref. 18). This interpretation also allows a connection between the turbulent kinetic theory in Ref. 9 and standard approaches to the nonequilibrium statistical mechanics of phase transitions.<sup>19</sup>

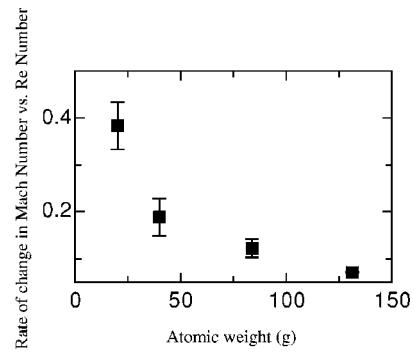


Fig. 10 Rate of change in Mach number with respect to Reynolds number vs atomic weight.

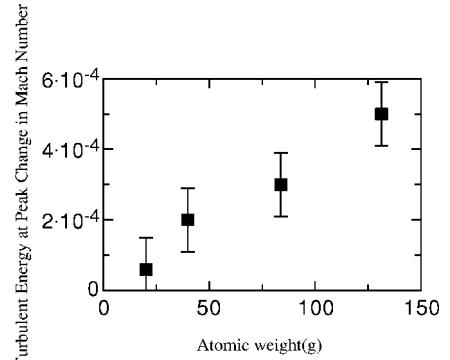


Fig. 11 Turbulent energy at peak percentage change in Mach number vs atomic weight.

Therefore, because Figs. 6-8 and 10 all suggest a defining relationship between the local temperature (as driven by the local Mach number), the local atomic weight, and the change in transport evidenced by Mach number enhancement from turbulence, the local turbulent energy associated with the defining conditions should have some special importance. In Fig. 11, the values of  $U_c$  vs atomic weight are shown, where  $U_c$  is the average value of  $U$  (over 80  $\mu$ s) of the turbulent fluctuation energy measured at  $\% \Delta M_c$  as defined earlier. Notice that, for increasing atomic weights, the turbulent energy associated with Mach number enhancement also increases.

#### IV. Summary

An investigation of the dynamics of shock waves in weakly ionized plasmas has been performed using a pressure ruptured shock tube and four different plasma species. The velocity of the shock wave is observed to increase when the shock wave traverses the plasma in all cases. The potential beneficial influence of a plasma species on shock wave drag increases with increasing atomic weight at fixed densities and degrees of ionization. If the evolution of  $\% \Delta M$  with Reynolds number  $Re$  is interpreted using a turbulent kinetic theory, then the dependence of  $\% \Delta M_c$  on atomic weight is at least qualitatively explained. Furthermore, if the change in the Mach number enhancement at  $U_c$  is interpreted using current approaches to phase transitions, then the appropriate sensitivity of  $U_c$  with atomic weight is observed, and thereby these data provide a first step toward a unified theory of turbulence.

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#### References

- Klimov, A. I., Koblov, A. N., Mishin, G. I., Serov, Y. L., and Yavor, I. P., "Shock Wave Propagation in a Glow Discharge," *Soviet Technical Physics Letters*, Vol. 8, No. 4, 1982, pp. 192-194.

<sup>2</sup>Kuo, S. P., and Bivolaru, D., "Plasma Effect on Shock Waves in a Supersonic Flow," *Physics of Plasmas*, Vol. 8, No. 7, 2001, pp. 3258-3264.

<sup>3</sup>Kuo, S. P., Kalkhoran, I. M., Bivolaru, D., and Orlick, L., "Observation of Shock Wave Elimination by a Plasma in a Mach-2.5 Flow," *Physics of Plasmas*, Vol. 7, No. 5, 2000, pp. 1345-1348.

<sup>4</sup>Gorshkov, V. A., Klimov, A. I., Koblov, A. N., Mishin, G. I., Khodataev, K. V., and Yavor, I. P., "Propagation of Shock Waves in a Glow Discharge Plasma in the Presence of a Magnetic Field," *Soviet Technical Physics Letters*, Vol. 29, No. 5, 1996, pp. 595-597.

<sup>5</sup>Ganguly, B. N., Bletzinger, P., and Garscadden, A., "Shock Wave Damping and Dispersion in Nonequilibrium Low Pressure Argon Plasmas," *Physics Letters A*, Vol. 230, 1997, p. 218.

<sup>6</sup>Appartaim, R., Thompson, K., and Johnson, J. A., III, "Shock Wave Dynamics in Plasmas for Hypersonic Drag Reduction," *22nd International Symposium on Shock Waves*, edited by R. Hillier, Imperial College, London, 1999, pp. 1123-1127.

<sup>7</sup>Howatson, A. M., *An Introduction to Gas Discharges*, 1965, pp. 160-179.

<sup>8</sup>Frisch, U., and Sulem, P. L., "Numerical Simulation of the Inverse Cascade in Two-Dimensional Turbulence," *Physics of Fluids*, 1984, pp. 1921-1923.

<sup>9</sup>Tsuge, S., "Approach to the Origin of Turbulence on the Basis of a Two-Point Kinetic Theory," *Physics of Fluids*, Vol. 17, Jan. 1974, pp. 22-33.

<sup>10</sup>Johnson, J. A., and Chen, S. C., "Reaction Rate Distortion in Turbulent Flow," *Physics Letters A*, Vol. 68, 1978, p. 141.

<sup>11</sup>Johnson, J. A., Johnson, L. E., and Lu, X., "Turbulence in a Reacting Contact Surface," *Physics of Fluids A*, Vol. 2, 1990, p. 2002.

<sup>12</sup>Johnson, J. A., and Santiago, J. P., "Turbulent Boundary Layer Treatment for Reacting Polymer Jets," *Polymer Communication*, Vol. 25, 1984, p. 34.

<sup>13</sup>Johnson, J. A., "Boundary-Layer Treatment for Turbulent Detonation Waves," *Applied Physics Letters*, Vol. 37, 1980, p. 275.

<sup>14</sup>De Silva, U., Gardner, A., and Johnson, J. A., III, "Turbulent Manipulation of Condensation at a Shock Tube's Contact Surface," *AIAA Journal*, Vol. 33, No. 2, 1995, pp. 368-370.

<sup>15</sup>Johnson, J. A., Lin, I., and Ramaiah, R., "Reduced Molecular Chaos and Flow Instability," *Stability in the Mechanics of Continua*, edited by F. H. Schroeder, Springer-Verlag, Berlin, 1982, pp. 318-329.

<sup>16</sup>Vincenti, W. G., and Kruger, C. H., *Introduction to Physical Gas Dynamics*, Wiley, New York, 1965, pp. 245-261.

<sup>17</sup>Greiner, W., Neise, L., and Stöcker, H., *Thermodynamics and Statistical Mechanics*, Springer-Verlag, 1995, pp. 245-435.

<sup>18</sup>Landau, L. D., and Lifshitz, E. M., *Statistical Physics*, Pt. 1, USSR Academy of Sciences, Moscow, 1980, pp. 446-450.

<sup>19</sup>Reichl, L. E., *A Modern Course in Statistical Physics*, 2nd ed., Wiley, New York, 1998, pp. 128-135, 753-765.

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